

On the mass and the density of stellar disk of M33.

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The disk surface density of the nearby spiral galaxy M33 is estimated assuming that it is marginally stable against gravitational perturbations. For this purpose we used the radial profile of line-of-sight velocity dispersion of the disk planetary nebulae obtained by Ciardullo et al. (2004). The surface density profile we obtained is characterized by the radial scalelength which is close to the photometrical one and is in a good agreement with the rotation curve of M33 and with the mass-to-light ratio corresponding to the observed color indices. However at the galactocentric distance $r > 7$ kpc the dynamical overheating of the disk remains quite possible. A thickness of the stellar disk of M33 should increase outwards. The dark halo mass exceeds the mass of the disk at $r > 7$ kpc. The obtained radial profile of the disk surface density and the radial gradient of O/H are used to calculate the effective oxygen yield Y_{eff} in the frame of the instantaneous recycling approximation. It is shown that Y_{eff} increases with radius which may indicate that the role of accretion of metal-poor gas in the chemical evolution of interstellar medium decreases outwards.

Key words: galaxies, galactic disks, gravitational stability of stellar disks

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1. Introduction

M33 is the late-type spiral galaxy of rather small size which belongs to the Local Group. Optical radius of the galaxy inside of $25^m/\square''$ isophote in B-band is close to 10 kpc. Due to the large angular size and the proximity to the Galaxy M33 is a perfect object for a detailed study of the density distribution of luminous and non-luminous matter. Of particular interest to this galaxy is the dark halo which includes a significant fraction of total mass within the optical borders. The indirect manifestation of the massive halo is the continued growth of the rotation curve at radial distances $r > 2h$, where h is a photometric disk radial scalelength (Persic et al., 1996). However the shape of the slowly rising rotation curve allows different ways to decompose the curve into the disk and dark halo components, which creates a problem of their mass ratio estimation. The attempts to decompose the galaxy into disk and halo components were done by Corbelli, Salucci (2000) and Corbelli (2003), which confirmed the domination of the dark halo within the optical radius. Assuming the disk radial scalelength to be equal to the K-band photometric scalelength, Corbelli (2003) came to conclusion that in the best fit model the rotation curve may be explained by the dark halo, which mass is about ten times higher than the mass of stellar disk, or at least five times higher than the disk baryonic mass (stars+gas) within two optical radii r_{25} . Independent disk mass estimation based on the condition for the existence of the wave spiral structure in M33 led to the dark halo mass fraction $M_h/M_d = 0.83$ inside the optical borders (see Athanassoula et al. 1987).

Later Ciardullo et al. (2004) estimated the disk mass of M33 by analyzing the measured velocity dispersion of planetary nebulae belonging to the stellar disk. By introducing some simplifying assumptions, these authors evaluated the vertical component of the velocity dispersion σ_z for the galactocentric distances $r = 1.2\text{--}8.5$ kpc to obtain the surface density of the equilibrium disk. The disk half thickness z_* (the scaleheight of planetary nebulae) was considered to be constant with radius (quite arbitrary it was assumed that $z_* = 175$ pc). The gravitational stability condition of a thin disk was superposed as an additional requirement. The model of the galaxy obtained by Ciardullo et al. (2004) differs much from that of Corbelli (2003): the disk radial scalelength found by Ciardullo et al. (2004) is twice as high as the optical K-band scale, which leads to conclusion that the mass-to-light ratio grows with galactocentric radius: M/L_V should be about five times higher in the periphery than inside of $r \approx 2$ kpc. The physical reason for this increase

remains enigmatic.

The alternative way to measure a disk mass can be based on the assumption that the disk self-gravity leads the disk to be close to the marginal stability state against the gravitational perturbations (this approach was first proposed by Zasov, 1985 and Morozov, Zasov, 1985 and later developed by Bottema, 1993). In general case, when marginal stability condition does not hold, the resulting density and mass estimates can be treated as the upper limits. There are some arguments supporting the idea that the disks of spiral galaxies are usually close to marginal stability. These are the absence of systematic deviation between the M/L values found for spiral galaxies under this assumption and from the photometrical models, and also the existence of correlation between the relative thickness of edge-on disks and the relative mass of their dark halo (see the discussion in Zasov et al. 2002, Zasov et al. 2011). The most promising way to construct models of marginally stable disks is to use the numerical 3D dynamical models. The numerical modeling of M33 disk confirms that the disk mass is low in comparison with the dark halo mass: if to admit that the photometrical radial scalelength is close to the disk density scalelength, then the velocity dispersion of planetary nebulae agrees with the model where the disk mass does not exceed one-third of the total mass inside of four radial scalelengths (Fridman, Khoperskov, 2011).

In the current paper we re-estimate the upper limit of the disk surface density using the line-of-sight velocity dispersion of planetary nebulae at different galactocentric distances r obtained by Ciardullo et al. (2004). As it follows from the analysis of stellar composition of M33, most stars of the galaxy have a large age exceeding 2-3 billion years (see e.g. Williams et al., 2009). As far as the planetary nebulae are associated with old stars (red giants), their velocity dispersion can be ascribed to stars which constitute the main body of the disk. Notice that the the mean velocity dispersion of the disk red giants is ~ 24 km/s (Hood et al. 2009), – in a good agreement with that of planetary nebulae.

Thus, the initial assumption is that the disk of M33 (at least inside of the radius covered by the velocity dispersion measurements) has the surface density which is close to the threshold for the gravitation stability (i.e. the disk is assumed to be in the equilibrium and marginally stable state). Below we use the modified analytical stability criterion and consider the disk radial scalelength and mass as a priori unknown parameters. The local disk density values found in this work were checked for compatibility with the rotation

curve of the galaxy and with the photometrically determined radial profile of M/L for stellar disk as well as with the available estimates of radial distribution of oxygen O/H . Following Ciardullo et al. (2004), we take the distance to the galaxy $D = 0.94$ Mpc

2. The stability criterion and the role of gas component

The constraints on the surface density of marginally stable disk can be found when the radial velocity dispersion of stars or planetary nebulae in a disk (that is the velocity dispersion along the radial coordinate r) c_r is known:

$$\sigma = \frac{c_r \kappa}{3.36 G Q_c}, \quad (1)$$

where $Q_c = c_r/c_T$ is the Toomre parameter of stability, c_T is the Toomre critical velocity dispersion, κ is epicyclic frequency defined as a function of the angular velocity Ω and its derivative:

$$\kappa = 2\Omega \cdot \sqrt{1 + (r/2\Omega)(d\Omega/dr)}. \quad (2)$$

The angular velocity at a given galactocentric distance was taken from the rotation curve of M33 obtained by Corbelli (2003). As the input data for stellar velocity dispersion we have used the radial profile of the observed line-of-sight velocity dispersion of the disk planetary nebulae given in Ciardullo et al., 2004 (see their Fig. 8).

Radial velocity dispersion is related to the line-of-sight dispersion c_{obs} through an obvious equation:

$$c_{obs}(r) = (c_z^2 \cdot \cos^2(i) + c_\phi^2 \cdot \sin^2(i) \cdot \cos^2(\alpha) + c_r^2 \cdot \sin^2(i) \cdot \sin^2(\alpha))^{1/2}, \quad (3)$$

where c_z , c_ϕ , c_r are the vertical, azimuthal and radial components, i is the disk inclination, α is the angle in the disk plane between the radius-vector of a planetary nebula and the major axis. To separate the components of the velocity dispersion one can introduce two additional conditions: $c_r = 2\Omega \cdot c_\phi / \kappa$ (Lindblad formula for the epicyclic approximation) and $c_z = m \cdot c_r$, where m is a coefficient assumed to be constant with radius. Both the available measurements of velocity dispersion of the galactic disks (Shapiro et al., 2003) and the results of numerical modeling (see e.g. Zasov et al., 2008) show that in most cases m lays within the range 0.4-0.7.

If the variation of the line-of-sight velocity dispersion along the disk major axis is known, then, taking $\alpha = 0$ in (3), we have:

$$c_r = c_{obs}(m^2 \cos^2(i) + (\kappa/2\Omega)^2 \sin^2(i))^{-1/2} \quad (4)$$

For the assumed inclination of the disk ($i=56^\circ$) the value of c_r weakly depends on c_z : the transition from $m=0.4$ to $m=0.7$ changes c_r by no more than 15%, so we restrict ourselves to the case of $m=0.4$ below. As it is followed from HI observations, the gaseous disk of M33 is warped in the outer part ($r > 30'$), and, as the result, its inclination and the position angle vary with the galactocentric distance (Corbelli, Schneider 1997). However this variation is significant only for the last point of the velocity dispersion profile of planetary nebulae. Therefore we assume the inclination and position angle of the disk to be fixed.

For the axisymmetric perturbations of marginally stable thin disk the Toomre parameter $Q_c = 1$. Non-axisymmetric perturbations require higher velocity dispersion to stabilize the disk while the finite thickness of the disk, by contrast, makes it more stable. In general case, the critical value of Q_c at a given radius depends in a complicated way on the geometrical and kinematical parameters of a disk as well as on the initial conditions of its dynamical evolution. A large number of 3D equilibrium models of marginally stable disks demonstrate that in the absence of massive bulge the critical value of Toomre parameter increases with the radius, and this variation may be approximated by parabola:

$$Q_c(r/h) = A_0 + A_1 \cdot (r/h) + A_2 \cdot (r/h)^2 \quad (5)$$

where $A_0 = 1.25$, $A_1 = -0.19$, $A_2 = 0.134$ and r/h is the radial distance expressed in the disk scalelength units (Khoperskov et al., 2003).

M33 is a gas-rich galaxy, where the ratio of gas to stellar surface densities exceeds 25% beyond $r \approx 4$ kpc, and the ratio of total masses of gaseous and stellar disks inside of the optical borders exceeds unity (see Corbelli (2003)). Hence the gas may play a significant role being a cold collisional component of the disk. The influence of a gas on a disk stability was investigated by different authors under certain simplifications. Here we use the results of analytical computation of stability of two-component models of a thin disk following Rafikov (2001) and Morozov, Khoperskov (2005).

Rafikov (2001) considered the Toomre parameters for purely gaseous and stellar components: $Q_* = \frac{\kappa C_*}{\pi G \sigma_*}$, $Q_{gas} = \frac{\kappa C_{gas}}{\pi G \sigma_{gas}}$ and the ratio $\nu = c_g/c_*$ of velocity dispersions of gas and stars. In his paper Rafikov presented the diagram $1/Q_{gas} - 1/Q_*$ for marginally stable stellar-gaseous disks for different values of ν . In turn, Morozov and Khoperskov derived the relationship between the radial velocity dispersion needed for stability of stellar-gaseous disk as a whole as a function of the ratios of velocity dispersions and surface densities of stellar and gaseous disk components.

Using these two papers, we found the coefficients corresponding to the observed ratios of the surface densities and velocity dispersions of stars and gas, which describe how the critical Toomre parameter should be changed to account for the gas in M33. These coefficients, lying in the range 1.1 - 1.9, were applied to the values of Q_c obtained for pure stellar disk from the equation (5). Thereby it is assumed that the corrections for the gas influence on the disk stability, obtained within the framework of simple analytical models, are valid for the disk of M33. The velocity dispersion of gas was assumed to be $c_{gas} = 8$ km/s independently on the radial distance. We used the radial distribution of gas following Corbelli (2003), allowing the contribution of helium and heavier elements.

3. The estimates of the disk surface density

Fig. 1 illustrates the surface density profiles of marginally stable stellar disk calculated from the available rotation curve and the observed velocity dispersion of planetary nebulae. The influence of gas on the stability is calculated following Rafikov, 2001) (model1) and Morozov, Khoperskov, 2005) (model 2). For comparison, the radial density profile obtained by Ciardullo et al. (2004) is also shown.

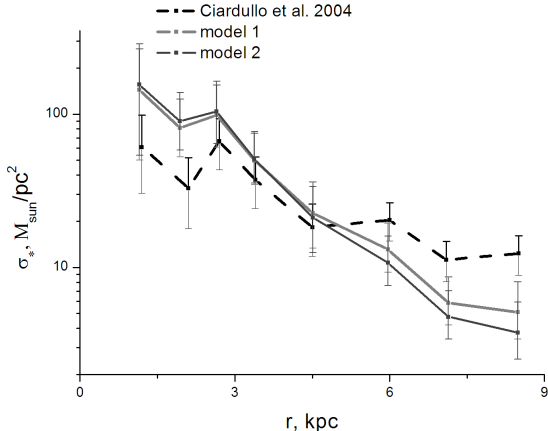


Fig. 1: Radial profiles of the stellar disk surface density. Dark- and light- gray lines demonstrate the profiles obtained for marginally stable disk taking into account the influence of gas component on the disk stability (models 1 and 2). Dashed line denotes the radial profile from Ciardullo et al. (2004). Here and in the following figures the error bars correspond to the errors of velocity dispersion measurements only (Ciardullo et al., 2004).

As it follows from Fig. 1, the difference between the models 1 and 2 is negligible, so we restrict ourselves to the model 1 below. At the same time these profiles differ significantly from that obtained by Ciardullo et al. (2004), where the scalelength of the surface density distribution is about $h \approx 4$ kpc, whereas in our models $h \approx 2$ kpc. The latter estimate is close to the photometrical radial scalelength of stellar disk: for K - and V -bands and the accepted distance to the galaxy we have $h_K = 1.45$ kpc, $h_V = 2.39$ kpc (Regan, Vogel, 1994, Baggett et al., 1998), $h_{3.6\mu m} = 1.9$ kpc (Seigar, 2011).

The inconsistency between the surface density profile of stellar disk from Ciardullo et al. (2004) with the photometrical profile can be a result of presumption of the constant thickness of the stellar disk. Indeed, the thickness of a disk is not always constant even in the first approximation. It can grow with the galactocentric distance, especially in the disk periphery (see e.g. de Grijs, Peletier, 1997). The radial variation of stellar disk half-thickness

$$z_*(r) \approx \frac{c_z^2(r)}{\pi G \sigma(r)},$$

calculated for $r < 4.5$ kpc, where the influence of dark halo on the stellar disk thickness may be negligible, is demonstrated in Fig. 2.

Unlike c_r , the stellar disk thickness estimated from the velocity dispersion measurements is sensitive to the choice of the badly known ratio $m = c_z/c_r$, therefore it cannot be unambiguously determined. Fig. 2 demonstrates the radial profiles of half-thickness $z_*(r)$ calculated for $m = 0.4$ (solid line) and $m = 0.7$ (dash-dotted line). As it follows from Fig. 2, the disk thickness increases rapidly to the periphery in both cases. Even for the low value $m = 0.4$ it exceeds that accepted by Ciardullo et al. (2004) beyond $r \approx 2.5$ kpc.

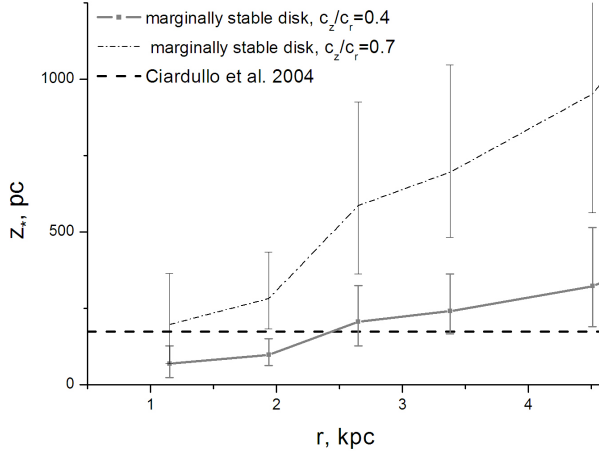


Fig. 2: The radial variation of the half-thickness of marginally stable stellar disk for $c_z/c_r=0.4$ (solid line) and $c_z/c_r=0.7$ (dash-dotted line). Dashed line shows the disk thickness adopted by Ciardullo et al. (2004).

In Fig. 3 we demonstrate the radial profiles of the local values of $M/L_K = \sigma_*/I_K$ (where σ_* is the stellar surface density), estimated for the marginally stable disk. The K-band surface brightness I_K was taken from Regan, Vogel (1994). Dashed line denotes M/L_K profile which was obtained from the photometric model of stellar population (Bell, de Jong, 2001) and the observed color indices: $(B - V)$ for the inner part (Guidoni et al. 1981) and $(H - K)$ for the outer part of the disk (Regan, Vogel 1994). A thin dotted line in Fig. 3a corresponds to the mass-to-light ratio corrected for internal dust extinction ($A_V \approx 0.25$, Verley et al. 2009). The internal extinction is negligible for M/L_K ratio found from the red color index $(H - K)$ (Fig. 3b).

Fig. 3 shows that the surface density of the disk, arising from the requirement of marginal gravitational stability, is in a good agreement with the photometry-based estimates with the exception of the most distant point ($r > 7$ kpc), where the dynamical overheating of the disk is quite possible. The real density of the disk at this point may be twice as low as for marginally stable condition.

The density profile of the disk obtained above was used to decompose the rotation curve, presented by Corbelli, 2003 (Fig. 4). The model contains four components: an exponential stellar disk with the scale length $h \approx 2$ kpc, a gaseous disk, corresponding to HI radial profile, a small bulge with effective radius $r_e = 2.2$ kpc (Regan, Vogel, 1994), which has a low mass and luminosity in comparison with the disk, and a pseudo-isothermal dark halo. Dark halo parameters were found by minimizing the deviation of the model rotation curve from the observed one. For

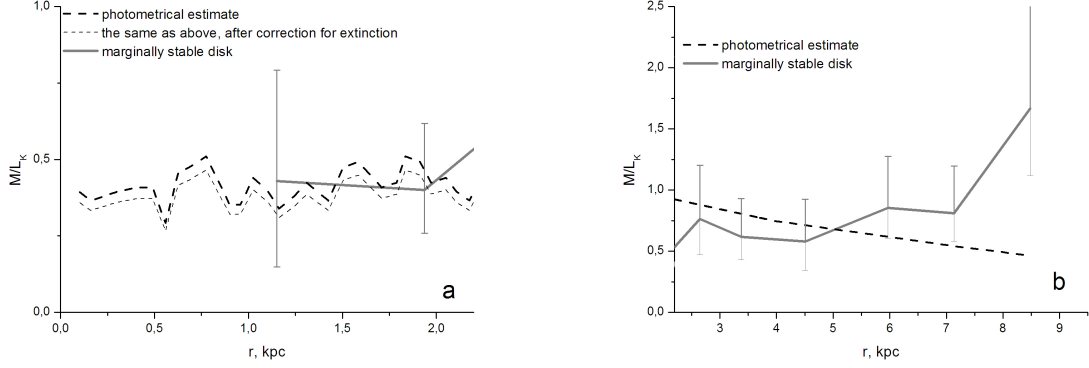


Fig. 3: Radial profiles of the mass-to-light ratio M/L_K of marginally stable disk (solid lines). The dashed lines correspond to the profiles based on the color indices and stellar population synthesis models (Bell, de Jong, 2001), obtained (a) for the inner part of the galaxy and $(B - V)$ profile from Guidoni et al. (1981); (b) for the outer part and $(H - K)$ profile taken from Regan, Vogel 1994). Thin dotted line is the profile corrected for the internal dust extinction $A_V = 0.25$ (Verley et al., 2009).

the dark halo mass distribution we found $v_a = 167$ km/s, $a = 7.5$ kpc, where v_a is the asymptotic velocity of the halo and a is its radial scale (the core radius).

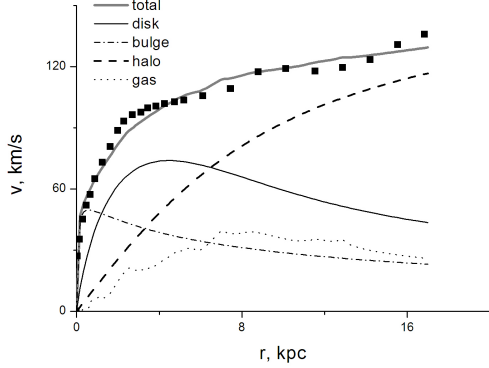


Fig. 4: The result of decomposition of the rotation curve taken from Corbelli, 2003 (squares) into the components: stellar and gaseous disks, bulge and dark halo, for the marginally stable disk model.

Fig. 4 demonstrates that the marginally stable disk with the parameters we found fits well into the observed rotation curve of the galaxy.

Radial variation of dark halo mass to luminous (stars+gas) mass ratio M_h/M_{vis} for the model described above is shown in Fig. 5. The masses of luminous and dark matter become equal at the galactocentric distance $r \approx 7$ kpc. Within $r = 17$ kpc the dark halo mass is about 5 times higher than the mass of luminous matter – in a good agreement with Corbelli (2003), where the best fit model of the rotation curve was used for a slightly lower disk scalelength $h \approx 1.45$ kpc. The halo-to-disk mass ratio inside $r = 4h \approx 8$ kpc for our model ($M_h/M_d \approx 2$) is lower than the ratio $M_h/M_d \approx 3$ given in the monograph by Fridman, Khoperskov, 2011. Note that

Athanassoula et al. (1987) gave even lower dark halo-to-disk mass ratio $M_h/M_d \approx 0.83$ inside of the optical radius using the spiral structure constraints, however the uncertainty of their method is rather high (about 0.3 dex).

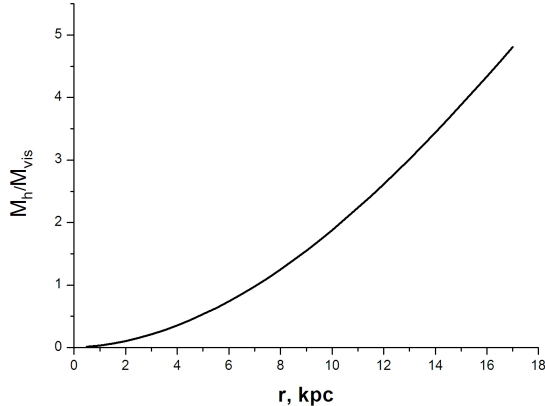


Fig. 5: The variation of the dark-to-luminous (stars+gas) mass ratio taken within the radius r .

4. Radial profile of oxygen effective yield in M33

Measurements of the gas metallicity, i.e. the abundance of heavy elements produced by stars, can give the additional information about the density of a disk and its chemical evolution. The important parameter, which characterizes the rate of chemical enrichment, is the yield of heavy elements Y , that is the mass fraction of a given chemical element (or all heavy elements) ejected into the interstellar medium by young stars. This parameter is connected with the nuclear evolution of stars of different masses and with the amount of the enriched gas that they lose. The observed abundance of any heavy element in the interstellar gas depends both on its yield and on those processes, which, parallel with star formation, affect the mass and density of the remaining gas (such as the blowing of gas out of the disk, the accretion of metal-poor gas, or radial gas flow). The observations allow to estimate the so-called effective yield Y_{eff} , found for the closed-box model of chemical evolution, where the following simplifications are assumed: 1) the system is closed: no gas inflow or outflow; 2) at the beginning of the disk evolution the gas metallicity is close to zero; 3) gas is chemically homogeneous at a given galactocentric distance; 4) stars eject the enriched gas immediately after their formation, that is the subsequent formation of stars takes place in the already enriched medium (the instantaneous recycling approximation); 5) the stellar initial mass function may be considered as not-evolved. In terms of this model the abundance of heavy elements Z is defined by the yield Y and the ratio of gas mass to total (gas and stars) mass of a disk:

$$Z = Y \cdot \ln(1/\mu), \quad (6)$$

where $\mu = \sigma_{gas}(r)/\sigma(r)$ is the gas mass fraction of a disk at a given galactocentric distance. The accretion of metal-poor gas as well as the outflow of gas leads to the decrease of Y_{eff} making it lower than the real yield Y (Edmunds, 1990). Hence, the simple model of chemical evolution of a galaxy is useful for testing the assumptions which it is based on.

The substantial fraction of the enriched matter ejected by stars belongs to oxygen, the most abundant element after H and He . The main producers of oxygen are the most massive short-lived stars, for which the condition of instantaneous recycling satisfies best of all. Thus it is very convenient to refer Y_{eff} to the effective yield of this particular element:

$$Y_{eff} = \frac{12(O/H)}{\ln(1/\mu)}, \quad (7)$$

where $12(O/H)$ is the mass fraction of oxygen. The known radial distributions of the disk surface density of M33 and O/H ratio allow to calculate the radial profile $Y_{eff}(R)$ and compare it to those predicted by different scenarios of the disk evolution. To estimate μ we used the radial distribution of hydrogen density taken from Corbelli (2003), applying the coefficient 1.33 for the total gas density.

Oxygen abundance O/H at different galactocentric distances of M33 was estimated by different authors. A significant spread of O/H values for HII regions, even for those that are at the same distance from the center, as well as some anomalies of oxygen abundance of giant HII regions in the central part of M33, led to a significant divergence of the existing estimates of radial gradient of O/H (see the discussion in Magrini et al., 2010). Furthermore, until recently there were only a small number of HII regions with the reliable measurements of O/H obtained from the gas temperature estimates T_e . Pilyugin et al. (2004) applied the developed P-method, which agrees with the T_e -based method, to the available spectra of HII regions in the sample of galaxies. For M33 they obtained the radial gradient of $O/H \approx -0.20 \text{ dex}/r_{25}$, which corresponds to -0.02 dex/kpc . The most complete list of O/H data for HII regions with known T_e was given later by Magrini et al. (2010). The gradient of O/H they found is $-0.033 \pm 0.008 \text{ dex/kpc}$ (for the distance accepted here). Below we use both gradients to calculate the radial distribution of Y_{eff} .

The results are demonstrated in Fig. 6a,b. Different lines correspond to the different radial density distribution. Thick solid line is related to the model of marginally stable disk. Dash-dotted and dashed lines correspond to the photometrically estimated density (see above) and to the model of Ciardullo et al. (2004). Thin horizontal line $Y_{eff} = 0.035$ denotes the estimate of real oxygen yield Y_O found by Pilyugin et al. (2007) from the gas metallicity in the central regions of spiral galaxies with the highest oxygen abundances. Close estimates of oxygen yield were obtained earlier by Bresolin et al. (2004) ($Y_O=0.032$) and Pilyugin et al. (2004) ($Y_O=0.027$).

It follows from Fig. 6, that the effective yield Y_{eff} increases from the center to the disk periphery for all radial profiles of the surface density we considered except the profile of Ciardullo et al. (2004), where Y_{eff} is approximately constant. Note however, that $Y_{eff} = \text{const}$ would mean that the gas accretion onto the galaxy is absent or at least it does not affect the present day abundance. It badly agrees with the current models of chemical evolution which include the external accretion as the necessary ingredient to account for the observed distribution of metallicity and density of gas in M33 (Magrini et al. 2007, 2010). Low Y_{eff} for the last point ($r \approx 8 \text{ kpc}$), if it is real, may be caused by the density overestimation due to the dynamical overheating of the disk at large radii, as it was noted above, because the photometry-based model does not reveal this feature. A reduction of Y_{eff} toward the center may indicate that the accretion of metal-poor gas is more essential for the inner part of the galaxy. In principle, it may be attributed either to the external accretion described by the current models of chemical evolution, or to the internal accretion, i.e. to the radial drift of less enriched gas toward the

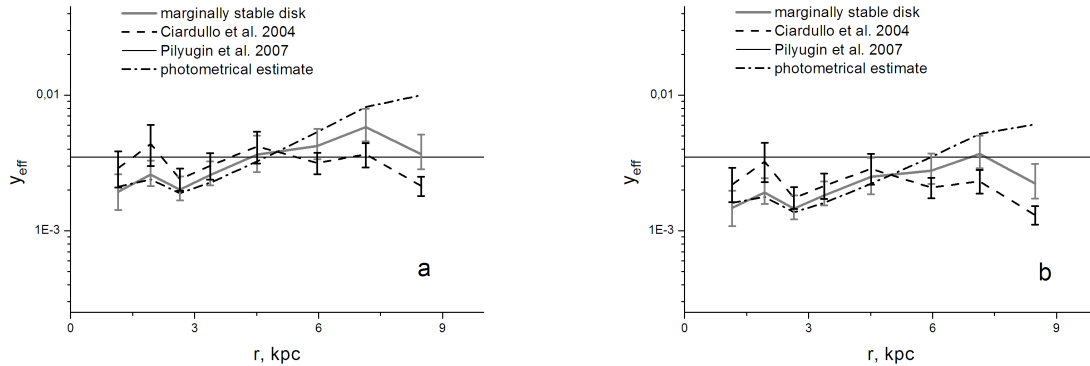


Fig. 6: Radial distribution of the effective yield calculated from the radial profile of O/H from Pilyugin et al., 2004 (a) and Magrini et al., 2010 (b). Solid line is for the marginally stable disk model, dash-dotted line corresponds to the photometrically obtained surface density of the disk, dashed line is for the disk surface density taken from Ciardullo et al. (2004). The horizontal line marks the oxygen yield Y_O found by Pilyugin et al. 2007 for the gas in the central regions of spiral galaxies with the highest oxygen abundance.

center. It is remarkable that the O/H data taken from Magrini et al. (2010), which are probably the most reliable, give the estimate of Y_{eff} at large radial distance close to Y_O , expected in the absence of accretion (the horizontal line in Fig. 6).

5. Conclusions

In summary, the assumption that the stellar velocity dispersion in the disk of M33 inside of $r \sim 6 - 7$ kpc is close to the minimal value needed to stabilize it, does not contradict either the photometrical profile of the disk, or the M/L ratio of stellar population, or the rotation curve of the galaxy, or the radial distribution of oxygen in the disk. It means that the disk of this galaxy within several radial scalelengths have not experienced a significant dynamical heating caused by interaction with nearby galaxies of Local Group or by minor merging events during its evolution. The existence of the massive dark halo is also confirmed: the dark halo mass begins to dominate over the disk mass starting with the radius of about 7 kpc. The radial distribution of the effective yield of oxygen, calculated in the frame of marginally stable disk model, decreases to the center. It supports the conclusion followed from the current chemical evolution models about the significant role of accretion in the chemical enrichment of interstellar medium in the inner disk. However these conclusions may not be extrapolated to the outer regions of the disk at $r > 7 - 8$ kpc where the dynamical overheating of the disk is quite possible and the dynamical and chemical evolution history may be more complicated.

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